Final exam

Patrick Loiseau

Game Theory, Fall 2015

2 hours, no document allowed except an A4 sheet of paper (both sides) with handwritten notes only.

Exercise 1 (~ 5 points)

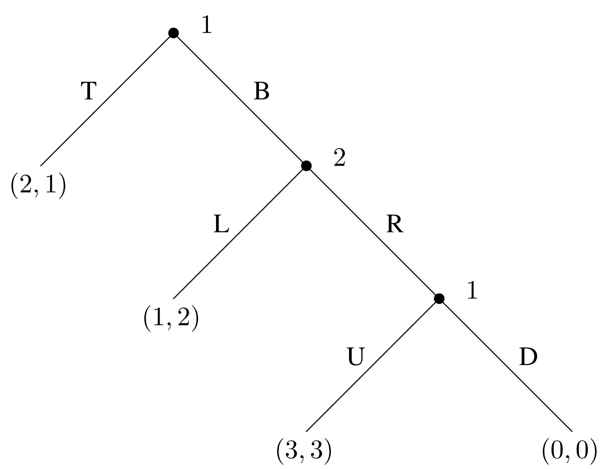
Consider the symmetric game with the following payoffs (in which $a \leq 2$ is a parameter):

	U	D
U	a, a	$3, 0$
D	$0, 3$	$2, 2$

1. Assume that $a > 0$. Find all Nash equilibria and all evolutionary stable strategies.
2. Assume that $a = 0$. Find all Nash equilibria in pure strategies and all pure evolutionary stable strategies.
3. Assume that $a < 0$. Find all Nash equilibria.

Exercise 2 (~ 6 points)

Consider the following game in extensive form. On the nodes where 1 (respectively 2) is written, player 1 (respectively 2) moves. For each outcome of the game, the first number represents the utility of player 1 and the second number the utility of player 2.



1. Apply backward induction.

2. Write the game in strategic form.

3. Find all pure Nash equilibria. Which ones are sub-game perfect?

4. Is there a pure Nash equilibrium which pareto dominates the other pure Nash equilibria?

Exercise 3 (~ 9 points)

We consider the following public good provision game. There are 2 players, each choosing the amount of money x_i ($i \in \{1, 2\}$) they will give to build a public good. We assume that each player has a maximum of 1 unit of money that he can give, so that $x_i \in [0, 1]$ for both players. Once the good is built, they receive a utility $h(G)$ from using it, where $G = x_1 + x_2$ is the total amount that was invested in the public good. We assume that $h(G) = KG^\alpha$, where $K \geq 0$ and $\alpha \in (0, 1)$ are constants. Each players utility is therefore

$$u_i(x_1, x_2) = K(x_1 + x_2)^\alpha - x_i \quad (i \in \{1, 2\}). \tag{1}$$

1. For a given value of $x_1 \in [0, 1]$, compute the best response of player 2. Give also the best response of player 1 to $x_2 \in [0, 1]$.

2. Draw the best response diagram in the three cases $K \in [0, \frac{1}{\alpha}]$, $K \in [\frac{1}{\alpha}, \frac{1}{\alpha}2^{1-\alpha}]$ and $K \geq \frac{1}{\alpha}2^{1-\alpha}$.

3. Give all Nash equilibria in pure strategy [hint: separate the cases $K \in [0, \frac{1}{\alpha}]$, $K \in [\frac{1}{\alpha}, \frac{1}{\alpha}2^{1-\alpha}]$ and $K \geq \frac{1}{\alpha}2^{1-\alpha}$].

4. Suppose that there is a social planner that can choose both x_1 and x_2 in order to maximize $u_1(x_1, x_2) + u_2(x_1, x_2)$. What values could he choose (give all possible solutions)? [hint: separate different regions depending on the value of K, but not the same regions as in the previous question.]

5. Compare the answer of question 4. to the Nash equilibria and comment.

6. Suppose now that $\alpha = 1$. Find all Nash equilibria in pure strategy.