Final exam

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Game Theory, Fall 2014

2 hours, no document allowed except an A4 sheet of paper (both sides) with handwritten notes only.

Questions (1 point each)

Reply to the following questions and justify your answer in 1 short paragraph

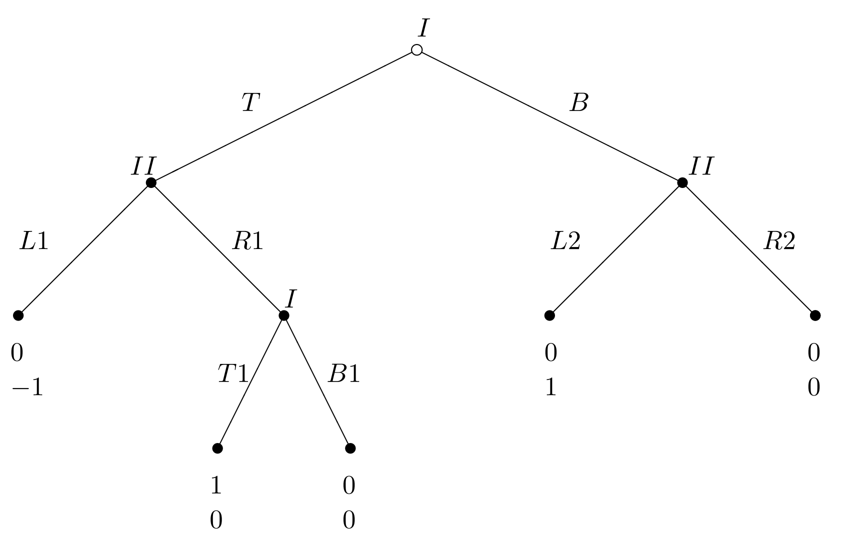
1. Compute the fully mixed strategies Nash equilibrium of the following game

	l	r
U	$11, 2$	$3, 7$
D	$6, 8$	$4, 2$

2. Is it true that, in a two players game where the players have two strategies each, the mixed strategies Nash equilibrium is always unique?

Exercise A (~ 7 points)

Consider the following game in extensive form. On the nodes where I (respectively II) is written, player 1 (respectively 2) moves. For each outcome of the game, the first number represents the utility of player 1 and the second number the utility of player 2.



1. Apply backward induction.
2. Write the game in strategic form.

3. Find all pure Nash equilibria. Which ones are sub-game perfect?
4. Is there a pure Nash equilibrium which pareto dominates the other pure Nash equilibria?

Exercise B (~ 7 points)

We consider the following public good provision game. There are 2 players, each choosing the amount of money x_i ($i \in \{1, 2\}$) she will give to build a public good. We assume that each player has a maximum of 1 unit of money that he can give, so that $x_i \in [0, 1]$ for both players. Once the good is built, each of them receives a utility $h(x_1, x_2) = kx_1x_2$ from using it, where $k \in [0, 2]$ is a constant, and pays a cost x_i^2. Each player utility is therefore

$$u_i(x_1, x_2) = kx_1x_2 - x_i^2 \quad (i \in \{1, 2\}). \tag{1}$$

1. For a given value of $x_1 \in [0, 1]$, compute the best response of player 2. Give also the best response of player 1 to $x_2 \in [0, 1]$.
2. Draw the best response diagram (hint: be careful to distinguish between $k \in [0, 2)$ and $k = 2$).
3. Give all Nash equilibria in pure strategies.
4. Suppose that there is a social planner that can choose both x_1 and x_2 in order to maximize $u_1(x_1, x_2) + u_2(x_1, x_2)$. What values could he choose?
5. Compare the answer of question 4. to the Nash equilibria and comment.

Exercise C (~ 4 points)

1. Two players. A pile of 8 chips. A move consists of removing 1, 2 or 3 chips. The player removing the last one wins (this is called a *Nim game*). Who wins?
2. Same game as before, but with 79 and 120 chips. Who wins?
3. Consider the *misère* version of the game with 8 chips, i.e. when the player removing the last one loses. Who wins? (hint: A winning stategy consists in forcing the opponent to take the last chip).