Final exam

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2 hours, no document allowed except an A4 sheet of paper (both sides) with handwritten notes only.

Questions (1 point each)

Say whether the following assertions are true or false and justify your answer in 1 short paragraph. In many cases, an example is sufficient.

1. A strictly dominated strategy can be played with positive probability in a Nash equilibrium strategy.
2. A weakly dominated strategy can be played with positive probability in a Nash equilibrium strategy.
3. A mixed strategy where one player plays 2 actions with positive probability can be a strict Nash equilibrium.
4. A symmetric Nash equilibrium in a symmetric 2 players game cannot be an evolutionary stable strategy if it is a weak Nash equilibrium.

Exercise A (~ 8 points)

We consider the following public good provision game. There are 2 players, each choosing the amount of money x_i ($i \in \{1, 2\}$) they will give to build a public good. We assume that each player has a maximum of 1 unit of money that he can give, so that $x_i \in [0, 1]$ for both players. Once the good is built, they receive a utility $h(G)$ from using it, where $G = x_1 + x_2$ is the total amount that was invested in the public good. We assume that $h(G) = K\sqrt{G}$, where $K \geq 0$ is a non-negative constant. Each players utility is therefore

$$u_i(x_1, x_2) = K\sqrt{x_1 + x_2} - x_i \quad (i \in \{1, 2\}). \tag{1}$$

1. For a given value of $x_1 \in [0, 1]$, compute the best response of player 2 (hint: be careful that it must be in $[0, 1]$). Give also the best response of player 1 to $x_2 \in [0, 1]$.
2. Draw the best response diagram in the three cases $K \in [0, 2]$, $K \in [2, 2\sqrt{2}]$ and $K \geq 2\sqrt{2}$.
3. Give all Nash equilibria in pure strategy (hint: separate the cases $K \in [0, 2]$, $K \in [2, 2\sqrt{2}]$ and $K \geq 2\sqrt{2}$).
4. Suppose that there is a social planner that can choose both x_1 and x_2 in order to maximize $u_1(x_1, x_2) + u_2(x_1, x_2)$. What values could he choose? (give all possible solutions)
5. Compare the answer of question 4. to the Nash equilibria and comment.

Exercise B (~ 8 points)

Consider the following two-player game:

	l	r
T	α, α	$0, \gamma$
B	$\gamma, 0$	β, β

where $\alpha > \beta > 0$ are fixed parameters and $\gamma > 0$ is a parameter which can take on different (positive) values. In this exercise we study the equilibria for different values of γ.

1. First assume that $\gamma > \alpha$.
   (a) Is there a strictly dominated strategy?
   (b) Find all Nash equilibria (pure and mixed).
   (c) What type of game is it?
2. Assume now that $\gamma \leq \alpha$.
   (a) Is there a strictly dominated strategy? Is there a weakly dominated strategy?
   (b) Find all Nash equilibria (pure and mixed).
   (c) For each Nash equilibrium (pure and mixed), say if it is strict or weak and under which condition on γ.
   (d) Among the pure strategy equilbria, say if one Pareto dominates the other.
   (e) Say which strategies (pure only) are evolutionary stable and under which condition on γ.
   (f) Suppose that $\gamma < \alpha$. Is the mixed strategy played at Nash equilibrium an evolutionary stable strategy?