Exercise sheet 4

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Exercise 1

1. Find all pure strategies and mixed strategies Nash equilibria of the following two-players game:

	a	b
A	$2,1$	$0,0$
B	$1,0$	$0,2$

Answer: Two pure strategies NE (A,a) and (B,b). An infinity of mixed strategies NE $((p,1-p),b)$ for any $p \in [0,2/3]$.

Exercise 2:

Consider the following two-persons game:

	l	r
U	$12,2$	$3,9$
D	$5,8$	$4,2$

1. Find all pure strategies and mixed strategies Nash equilibria.

Answer: $\left(\left(\frac{6}{13},\frac{7}{13}\right),\left(\frac{1}{8},\frac{7}{8}\right)\right)$.

2. Assume now that $u_2(D,l)$ is reduced from 8 to 6. Find all pure strategies and mixed strategies Nash equilibria.

Answer: $\left(\left(\frac{4}{11},\frac{7}{11}\right),\left(\frac{1}{8},\frac{7}{8}\right)\right)$.

3. Compare the strategies of player 1 and 2 in the mixed strategy Nash equilibria of questions 1. and 2. Comment.

Answer: Reducing the utility of the second player, we do not modify her optimal strategies but the ones of the other player.

Exercise 3:

Suppose that player 1's car is not working properly: it lacks power. He does not know whether it needs a small engine cleaning or a major repair (say, a new engine). The probability that it needs a new laser is ρ. At his local garage, he finds that a new engine costs L, while a cleaning costs C ($L > C$). He knows that the expert at the garage, player 2, gets the same profit π, if she charges him for a new engine and indeed fixes the engine, or if she charges him for a cleaning and indeed just cleans it. But she can make more profit, $\Pi > \pi$ if she charges him for a new engine but in fact (secretly) just cleans it. If it only needed a cleaning anyway, then she will get away with this, but she knows she will get sent to jail if she only cleans it when it needed a new engine. The expert is very good at her job, so she knows which is needed.

1. Explain why player 1 should always believe player 2 when she says it just needs a cleaning but why he might be skeptical if she says it needs a new laser.

   Answer: No game yet. If needs engine, player 2 will say so.

Player 1 can reject the local expert's advice and get a second opinion from a consultant who never lies. Assume however that, if he does so, he must accept the second expert's advice and accept new repair costs $L' > L$ or $C' > C$. The game is then:

	Honesty	Dishonesty
Always accept advice	$-\rho L - (1-\rho)C, \pi$	$-L, \rho\pi + (1-\rho)\Pi$
Reject if told 'new engine'	$-\rho L' - (1-\rho)C, (1-\rho)\pi$	$-\rho L' - (1-\rho)C', 0$

2. Explain the terms in the payoff matrix.

3. Assume that $L > \rho L' + (1-\rho)C'$. Is there a pure strategy Nash equilibrium?

   Answer: No.

4. Find the mixed strategy Nash equilibrium (as a function of the parameters).

   Answer: $p = \frac{\pi}{\Pi}$ **and** $q = \frac{L-\rho L'-(1-\rho)C'}{L-\rho L-(1-\rho)C'}$

5. As we increase the cost of repair at the local garage L, what happens to the equilibrium probability that the expert chooses 'honest'? What happens to the equilibrium probability that player 1 chooses 'Reject if told 'new engine"? Comment.

   Answer: When we increase L**,** q **increases as well, while** p **is not affected by** L**.**

6. As we increase the profit from lying Π, what happens to the equilibrium probability that the expert chooses 'honest'? What happens to the equilibrium probability that player 1 chooses 'Reject if told 'new engine"? Comment.

   Answer: When we increase Π**,** q **is not affected, while** p **decreases.**

7. It has been said that, in America, when people go to the doctor, they never think they have a cold: they think they have 'mono'. Assuming this is true, why might we expect doctors in America often to act dishonestly? [Hint: think about how the parameter ρ affects the equilibrium in the above model].

   Answer: ρ does not affect p, but q. When we assume ρ bigger, we think that 2 will act honestly, so we are more incentivated to accept.