Exercise sheet 4

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Exercise 1

1. Find all pure strategies and mixed strategies Nash equilibria of the following two-players game:

	a	b
A	2, 1	0, 0
B	1, 0	0, 2

Exercise 2:

Consider the following two-persons game:

	l	r
U	12, 2	3, 9
D	5, 8	4, 2

1. Find all pure strategies and mixed strategies Nash equilibria.

2. Assume now that $u_2(D, l)$ is reduced from 8 to 6. Find all pure strategies and mixed strategies Nash equilibria.

3. Compare the strategies of player 1 and 2 in the mixed strategy Nash equilibria of questions 1. and 2. Comment.

Exercise 3:

Suppose that player 1's car is not working properly: it lacks power. He does not know whether it needs a small engine cleaning or a major repair (say, a new engine). The probability that it needs a new laser is ρ. At his local garage, he finds that a new engine costs L, while a cleaning costs C ($L > C$). He knows that the expert at the garage, player 2, gets the same profit π, if she charges him for a new engine and indeed fixes the engine, or if she charges him for a cleaning and indeed just cleans it. But she can make more profit, $\Pi > \pi$ if she charges him for a new engine but in fact (secretly) just cleans it. If it only needed a cleaning anyway, then she will get away with this, but she knows she will get sent to jail if she only cleans it when it needed a new engine. The expert is very good at her job, so she knows which is needed.

1. Explain why player 1 should always believe player 2 when she says it just needs a cleaning but why he might be skeptical if she says it needs a new laser.

Player 1 can reject the local expert's advice and get a second opinion from a consultant who never lies. Assume however that, if he does so, he must accept the second expert's advice and accept new repair costs $L' > L$ or $C' > C$. The game is then:

	Honesty	Dishonesty
Always accept advice	$-\rho L - (1-\rho)C, \pi$	$-L, \rho\pi + (1-\rho)\Pi$
Reject if told 'new engine'	$-\rho L' - (1-\rho)C, (1-\rho)\pi$	$-\rho L' - (1-\rho)C', 0$

2. Explain the terms in the payoff matrix.

3. Assume that $L > \rho L' + (1 - \rho)C'$. Is there a pure strategy Nash equilibrium?

4. Find the mixed strategy Nash equilibrium (as a function of the parameters).

5. As we increase the cost of repair at the local garage L, what happens to the equilibrium probability that the expert chooses 'honest'? What happens to the equilibrium probability that player 1 chooses 'Reject if told 'new engine"? Comment.

6. As we increase the profit from lying Π, what happens to the equilibrium probability that the expert chooses 'honest'? What happens to the equilibrium probability that player 1 chooses 'Reject if told 'new engine"? Comment.

7. It has been said that, in America, when people go to the doctor, they never think they have a cold: they think they have 'mono'. Assuming this is true, why might we expect doctors in America often to act dishonestly? [Hint: think about how the parameter ρ affects the equilibrium in the above model].