

# Exercise sheet 2

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## Exercise 1: Splitting the dollar

Two players bargain over how to split \$10. Each player  $i \in \{1, 2\}$  chooses a number  $s_i \in [0, 10]$  (which does not need to be an integer). Each player's payoff is the money he receives. We consider two allocation rules. In each case, if  $s_1 + s_2 \leq 10$ , each player gets his chosen amount  $s_i$  and the rest is destroyed.

1. In the first case, if  $s_1 + s_2 > 10$ , both players get zero. What are the (pure strategy) Nash equilibria?
2. In the second case, if  $s_1 + s_2 > 10$  and  $s_1 \neq s_2$ , the player who chose the smallest amount receives this amount and the other gets the rest. If  $s_1 + s_2 > 10$  and  $s_1 = s_2$ , they both get \$5. What are the (pure strategy) Nash equilibria?
3. Now suppose that  $s_1$  and  $s_2$  must be integers. Does this change the (pure strategy) Nash equilibria in either case?

## Exercise 2: Partnership game

Two partners jointly own a firm and equally share its revenue. Each partner  $s_i$  chooses how much effort to put into the firm. We denote by  $s_i$  the effort of partner  $i$  (say, in hours) and assume that the action set of each partner is  $S_i = [0, 4]$ . The firm revenue is given by  $4(s_1 + s_2 + bs_1s_2)$ , where  $b \in (0, 1/4]$  is a parameter measuring the synergy of the two partners. We assume that the cost supported by a partner equals the square of its effort  $s_i$  (notice that the cost of providing another unit of effort is therefore increasing in the amount of effort already provided). The payoffs are then

$$u_1(s_1, s_2) = \frac{1}{2}[4(s_1 + s_2 + bs_1s_2)] - s_1^2$$
$$u_2(s_1, s_2) = \frac{1}{2}[4(s_1 + s_2 + bs_1s_2)] - s_2^2$$

1. Find the best response of partner 1 to an effort  $s_2$  of partner 2.
2. Find the best response of partner 2 to an effort  $s_1$  of partner 1. [Hint: look at the problem's symmetry.]
3. On a single graph with  $s_1$  on the  $x$ -axis and  $s_2$  on the  $y$ -axis, plot the two best responses. (Such a plot is called a best-response diagram.)
4. Find the Nash equilibrium and show its position on the diagram.
5. What are the efforts  $(s_1, s_2)$  that would maximize the total net profit of the firm (i.e., the sum of the utility of each partner)? How does it compare to the Nash equilibrium?